SYNCHRONIZING AUTONOMOUS CHAOTIC CIRCUITS USING BANDPASS FILTERED SIGNALS

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ABSTRACT

The same broad-band properties that make chaos interesting as a potential communications carrier also cause trouble when chaotic signals are transmitted through real communications channels. Phase and amplitude distortion of a chaotic signal can ruin the ability of a chaotic receiver to synchronize to an incoming chaotic signal. We show that it is possible to use narrow band filters with the chaotic signal to reduce its bandwidth, decreasing the effects of channel distortion, and still synchronize a chaotic receiver to the incoming chaotic signal.

INTRODUCTION

There are many approaches to communication with chaotic signals [1-6], but the issue of how to successfully transmit a chaotic signal with minimum distortion through a communications channel has yet to be resolved. Adaptive techniques for reversing channel distortion have been proposed [7, 8], as well as methods to create narrow band chaotic transmitters [9]. Our work here takes a non-adaptive approach: we pass a broad-band chaotic signal through a narrow band filter, either before or after transmission through the channel. We pass an identical signal from the chaotic receiver through an identical filter, and take the difference between the filtered transmitter signal and the filtered receiver signal. The difference signal is fed back into the receiver. when the transmitter and receiver are synchronized, the difference signal is zero.

SYNCHRONIZATION TECHNIQUES

In general, we may express our drive and response systems in the form

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= F(\mathbf{x}) & \frac{d\mathbf{x}'}{dt} &= F(\mathbf{x}') + \mathbf{b}(g_i - g'_i) \\ u &= \mathbf{k}\cdot\mathbf{x} & u' &= \mathbf{k}\cdot\mathbf{x}' \\ \frac{d\mathbf{f}}{dt} &= G(\mathbf{f}, u) & \frac{d\mathbf{f}'}{dt} &= G(\mathbf{f}', u') \end{aligned} \qquad (1)$$

where $\mathbf{x}$ is the drive system state vector, $\mathbf{x}'$ is the response system state vector, $\mathbf{k}$ and $\mathbf{b}$ are constant vectors (suggested by the work of Peng et al [10]), u is a scalar, G is a dynamical system, and g_i is a signal taken from the dynamical system G. We make a linear combination u of signals from the drive system $F(\mathbf{x})$ and drive the dynamical system G with u. We then take a signal from G, such as g_i, and transmit it to the response system. We set up the response system in the same way, then multiply the difference (g_i' - g_i) by the vector $\mathbf{b}$ and add it to the response vector field. We find that if the response system (including G) has all Lyapunov exponents less than zero, the response will synchronize to the drive. In this work, we use bandpass filters for the dynamical system G, although other dynamical systems may work. A bandpass filter passes only a certain band of frequencies from the input signal.

NUMERIC EXPERIMENTS

As a numerical example, we link two Lorenz systems [11] through a bandpass filter. For our Lorenz example, the vector field F was given by $dx_1/dt = 16(x_2 - x_1)$, $dx_2/dt = -x_1x_3 + 45.92x_1 - x_2$, and $dx_3/dt = x_1x_2 - 4x_3$. The scalar u was $u = k_1x_1 + k_2x_2 + k_3x_3$. The filter G was described by

$$\frac{dg_1}{dt} = -\frac{2}{R_1C}g_1 - \left(\frac{1}{2R_2C}\right)\left(\frac{1}{R_3C} - \frac{1}{R_1C}\right)g_2 - \left(\frac{1}{R_1C}\right)\frac{du}{dt}$$

$$\frac{dg_2}{dt} = g_1 \tag{2}$$

At the response system, we took the difference (g_2' - g_2), multiplied by a vector **b** = (b_1, b_2, b_3) and added the result to the response vector field.

Equation (2) represents a second order bandpass filter [12]. The center frequency f_c is passed with a gain of 1, while other frequencies are attenuated by an amount that increases as they become farther from f_c. The constants were C = 1, R_1 = 3.183, and R_2 = 6.366. R_3 was used to vary the center frequency of the filter, so for a center frequency of f_c, $R_3 = R_1/(-1 + 4(\pi f_c)^2 R_1 R_2)$. For these parameters, the Q factor of the filter was 20 (the Q factor is the center frequency f_c divided by the bandwidth). The center frequency f_c of the filter was varied between 0.1 and 10. Numerical integration was carried out by a 4-th order Runge-Kutta integration routine [13]. The components of **k** and **b** [10] were chosen by a numerical minimization routine to make the largest Lyapunov exponent for the response system less than zero.

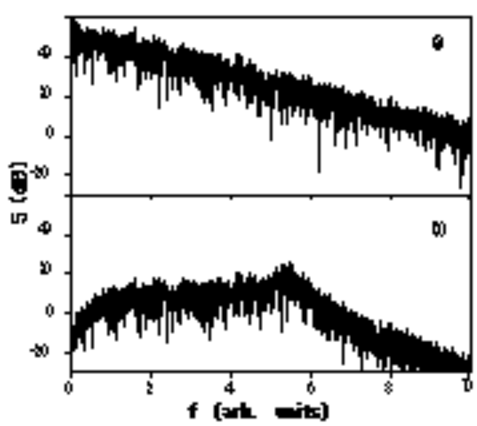

Figure 1. (a) Power spectrum of transmitted signal u before filtering. (b) Power spectrum of transmitted signal u after filtering.

Figure 1(a) shows the power spectrum of the signal u for the Lorenz system. Figure 1(b) shows the power spectrum of the signal g_2 the filtered version of u, for a center frequency f_c = 5.44. The components of **k** and **b** were k_1 = 273.0212, k_2 = 23.26557, k_3 = 16.24705, b_1 = 18.93643, b_2 = 20.51921, and b_3 = -3.04397. For these parameters, the largest Lyapunov exponent for the response system was -4.95. Figure 2 shows the synchronization of the response system to the drive system. We found that with the above parameters, the response system was stable for f_c ranging from about 1 to about 9.

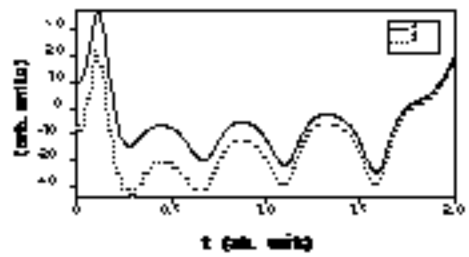

Figure 2. Synchronization of Lorenz systems when response is driven through a filter. Solid line is the drive system and dotted line is the response system.

CIRCUIT EXAMPLE

We also built a set of electronic circuits that could be synchronized through a filter. Our drive and response circuits were piecewise linear circuits [14] whose attractors resembled the Rossler attractor. We filtered out all but the central peak in the transmitted signal spectrum (Fig. 3(a)) by bandstop filtering the signal u from the Rossler drive circuit and subtracting from the unfiltered signal. We found that this arrangement was more stable for our circuits than a bandpass filter.

Our drive circuit vector field was described by $dx_1/dt = -\gamma\ (0.05x_1 + 0.5x_2 + x_3)$, $dx_2/dt = -\gamma\ (-x_1 - 0.11\ x_2)$, and $dx_3/dt = -\gamma\ (x_3 + h(x_1))$, where $h(x) = 0$ if x 3 and $h(x) = 15(x - 3)$ if $x > 3$. The time factor γ was $10^4\ s^{-1}$.

The filter G was described by

$$\frac{dg_1}{dt} = -\left(\frac{1}{RC}\right)\left(\frac{3g_1}{1+\alpha} + g_2 + \frac{\beta}{1+\alpha}u\right) - \frac{\beta}{1+\alpha}RC\frac{d^2u}{dt^2} \qquad (3)$$

$$\frac{dg_2}{dt} = \frac{1}{RC}g_1$$

$$g_f = u + g_2$$

where the narrow band output signal was g_f. The filter Q was given by $(\alpha + 1)/3$, and the filter gain was $-\beta/(1+\alpha)$. The Q factor was set to 7 ($\alpha = 20$) and the gain to -1 ($\beta = 1+\alpha$.). The filter center frequency f_c (the frequency at which the bandstop output was zero) was $1/(2\pi RC)$. We set the center frequency to coincide with the main frequency peak in the spectrum of the signal u from the circuit, at 1145 Hz. Figure 3(a) shows the unfiltered power spectrum of u, while Fig 3(b) is the power spectrum of the filtered signal g_f. We transmitted the signal g_f to the response circuit.

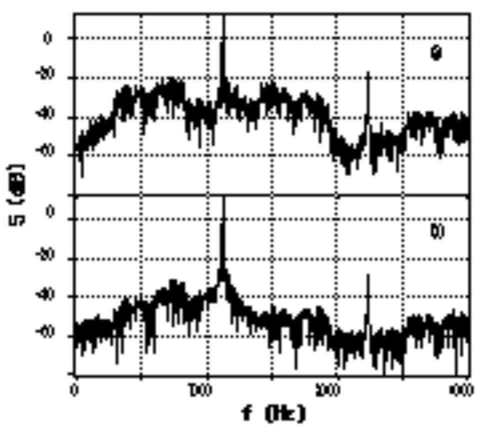

Figure 3. (a) Power spectrum before filtering of the transmitted signal u from the drive circuit. (b) Power spectrum of the filtered drive signal.

The response circuit was piecewise linear, so we were able to estimate the stability of the response circuit by finding a Jacobian for the case g(x) = 0. This Jacobian was constant, so we used the largest real part of the eigenvalues of the Jacobian as an estimate of the stability of the response circuit. We varied the components of **k** and **b** to find a stable response system.

The response circuit was stable for $k_1 = -1.9$, $k_2 = 1.1$, $k_3 = 1$, $b_1 = 1$, $b_2 = 1$, and $b_3 = 1$. The largest real part of the eigenvalues for the response circuit was -1,170 . Figure 4 shows v from the response circuit vs. u from the drive circuit, showing synchronization.

One might ask why a narrow band filter passes enough information to synchronize a response circuit. We may divide the chaotic motion into motion on a synchronization manifold (where the systems are synchronized) and motion transverse to the synchronization manifold. Hunt and Ott [15, 16] have stated that one gets the optimal average of any smooth function of a system state by averaging over a low period orbit, so if the averages over several low period orbits of the Lyapunov exponents transverse to the synchronization manifold are negative, we should see synchronization [17, 18]. For the piecewise linear Rossler circuit, all of the low period unstable orbits have a large spectral component at the main peak in the Rossler spectrum, so if we filter at this peak frequency, we can stabilize all of the low period orbits at once.

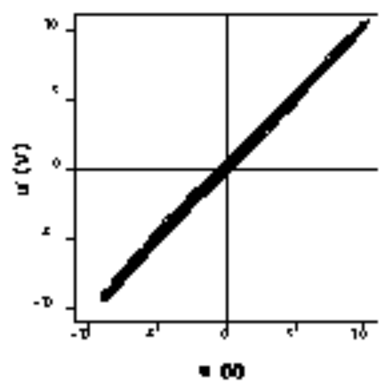

Figure 4. Signal u' from the response circuit vs. signal u from the drive circuit, showing synchronization.

The periodic orbits for the Lorenz system contain many different frequencies. Although the global Lyapunov exponents for the Lorenz response system above are always negative, the local Lyapunov exponents are sometimes negative and sometimes positive. We are able to make the average Lyapunov exponent negative because we are stabilizing one or more low period orbits, which dominate the average. We added 1% random noise to the Lorenz simulation and saw no evidence bursting away from synchronization [17, 18].

CONCLUSIONS

Using a narrow band signal to synchronize broad band systems has some obvious advantages for applications in communications.

Reduced bandwidth means that the transmitted signal will suffer less distortion. Filtering the transmitted signal at the receiver will remove much of the noise picked up in transmission. One could even synchronize multiple response systems to the same chaotic signal filtered at different frequencies. By comparing the different response systems, one might be able to reduce the effects of frequency dependent noise.

Adding filters to synchronized chaotic systems does bring some loss of stability, so the filtered systems will take longer to synchronize and be less robust to noise that is not filtered out. One may understand this loss of stability by considering the filtering as a convolution of a time series with some filter function. The narrower the passband of the filter, the longer the time over which the filter averages the incoming signal. Long time averages mean that the filter cannot respond quickly to changes in the incoming signal, so the response system is less stable. Rulkov [9] has described an alternate method which avoids this stability problem by designing narrow-band chaotic systems.

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